Exercise 23A: Solutions

1 a

$$\begin{aligned} & x \in (A \cap B)' \\ \Leftrightarrow\ & x \notin A \cap B \\ \Leftrightarrow\ & x \notin A \text{ or } x \notin B \\ \Leftrightarrow\ & x \in A' \cup B' \end{aligned}$$

b

$$\begin{aligned} & x \in (A \cup B) \cap (A \cup B') \\ \Leftrightarrow\ & x \in A \text{ or } x \in B \text{ and} x \in A \text{ or } x \in B' \\ \Leftrightarrow\ & x \in A \text{ and } x \in B \text{ or } x \in B' \\ \Leftrightarrow\ & x \in A \end{aligned}$$

c

$$\begin{aligned} & x \in (A \cap B) \cup (A \cap B') \\ \Leftrightarrow\ & x \in A \cap (B \cup B \\ \Leftrightarrow\ & x \in A \cap \xi \\ \Leftrightarrow\ & x \in A \end{aligned}$$

d

$$\begin{aligned} & x \in (P \cap Q)' \cup (P \cap Q) \\ \Leftrightarrow\ & x \notin (P \cup Q) \text{ or } x \in (P \cap Q) \\ \Leftrightarrow\ & (x \notin P \text{ and } x \notin Q) \text{ or } x \in (P \cap Q) \\ \Leftrightarrow\ & (x \notin P \text{ and } x \notin Q \text{ or } x \in P) \text{ and } (x \notin P \text{ and } x \notin Q \text{ or } x \in Q) \\ \Leftrightarrow\ & x \in (P \cup (P' \cap Q')) \cap (Q \cup (P' \cap Q')) \\ \Leftrightarrow\ & x \in (P' \cup Q) \cap (Q' \cup P) \end{aligned}$$

2 a

$$\begin{aligned} & x \in P \backslash (Q \backslash R) \\ \Leftrightarrow\ & x \in P \text{ but } x \notin (Q \backslash R) \\ \Leftrightarrow\ & x \in P \backslash Q \cup (P \cup R) \end{aligned}$$

b

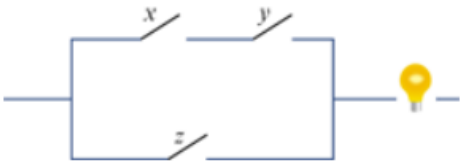
$$\begin{aligned} & x \in P \cap (Q \backslash R) \\ \Leftrightarrow\ & x \in P \cap Q \text{ and } x \notin (P \cap R) \\ \Leftrightarrow\ & x \in (P \cap Q) \backslash (P \cap R) \end{aligned}$$

3 a $(A \cap \emptyset) \cup (A \cup \xi) = \xi$

b If $A \cup B = \xi$, then $A' \cap B = A'$.

c $A \cup B \supseteq A \cap B$

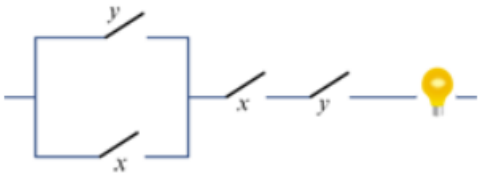
4 a i



ii

x	y	z	$x \wedge y$	$(x \wedge y) \vee z$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

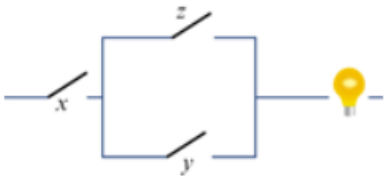
b **i**



ii

x	y	$x \vee y$	$x \wedge y$	$(x \vee y) \wedge (x \wedge y)$
0	0	0	0	0
0	1	1	0	0
1	0	1	0	0
1	1	1	1	1

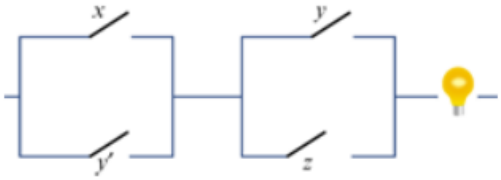
c **i**



ii

x	y	z	$y \vee z$	$x \wedge (y \vee z)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

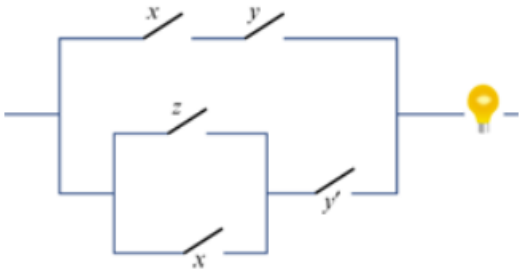
d **i**



ii

x	y	z	$a = x \vee y'$	$b = y \vee z$	$a \wedge b$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

5 a



b

x	y	z	$a = x \wedge y$	$b = (z \vee x) \wedge y'$	$a \vee b$
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	0	1